

Refresh memory:

Binomial Distribution:

- S** success/failure
- N** fixed number "n" of observations
- I** each observation is independent
- P** each observation has same probability "p"

Binomial variable "X" counts number of **successes (k)** in the fixed number of trials

Examples:

How many free throws will Corinne make if she makes 75% of her throws and shoots 12 in a game?

How many children out of 5 in one family will have type "O" blood if there is a 25% chance of each child getting it?

8.2 Geometric Distributions

- S** success/failure
- H** how many observations until success?
- I** each observation is independent
- P** each observation has same probability "p"

Random variable "X" counts number of **trials (n)** needed to reach the first success.

Examples:

How many times will I need to flip a coin until I get a head?

How many times will I roll a die before getting a 2?

How many times will Corinne attempt a three point shot until she makes a basket?

*notice in each example, we are counting trials (n), not successes (k).

Example 8.15 (p.465)

An experiment has you rolling a single die. We want to roll a 2, which we will call a success. The random variable X = the number of trials until a 2 occurs.

Check for SHIP: (2 = success, all others = failure, how many rolls until a 2, die rolls are independent, each roll has a 1/6 probability)

$$X = 1: P(X=1) = P(\text{success on 1}^{\text{st}} \text{ roll}) = 1/6$$

$$\begin{aligned} X = 2: P(X=2) &= P(\text{success on 2}^{\text{nd}} \text{ roll}) \\ &= P(\text{failure on 1}^{\text{st}} \text{ and success on 2}^{\text{nd}}) \\ &= P(\text{failure on 1}^{\text{st}}) \times P(\text{success on 2}^{\text{nd}}) \quad \text{*independent} \\ &= (5/6)(1/6) \end{aligned}$$

$$\begin{aligned} X = 3: P(X=3) &= P(F \text{ on 1}^{\text{st}}) \times P(F \text{ on 2}^{\text{nd}}) \times P(S \text{ on 3}^{\text{rd}}) \\ &= (5/6)(5/6)(1/6) \end{aligned}$$

$$P(X=n) = (5/6)^{n-1}(1/6)$$

General Rule for calculating Geometric Probabilities:

$$P(X=n) = (1-p)^{n-1}(p)$$

Creating a probability distribution table

use the dice experiment:

X	1	2	3	4
$P(X)$	1/6	(5/6)(1/6)	(5/6)(5/6)(1/6)	(5/6)(5/6)(5/6)(1/6)
$P(X)$	p	qp	q^2p	q^3p

with the calculator, just multiply each $P(X)$ by $P(\text{failure})$

The terms of $P(X)$ are a geometric sequence, since each term is being multiplied by the same constant to get to the next term

You don't need to know this, but it's interesting...

Theoretically, to be a valid distribution, the probabilities have to add to 1.

$$S = p + (1-p)p + (1-p)^2p + (1-p)^3p + \dots$$

$$\text{let } 1-p = q$$

$$S = p + qp + q^2p + q^3p + \dots$$

multiply both sides by q (because we can)

$$qS = qp + q(qp) + q(q^2p) + q(q^3p) + \dots$$

$$qS = qp + q^2p + q^3p + q^4p + \dots \quad (\text{simplify})$$

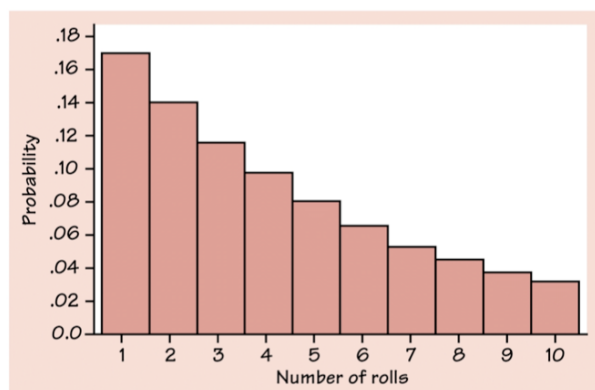
$$S = p + qp + q^2p + q^3p + \dots \quad (\text{remember what } S = \text{ above})$$

$$S - qS = p \quad (\text{subtract the two equations})$$

$$S(1-q) = p \quad \dots \quad S = p/(1-q) \quad \text{or} \quad p/p = 1 \quad (\text{work algebra magic})$$

Be able to graph a geometric distribution: (histogram)

X	1	2	3	4
P(X)	1/6	(5/6)(1/6)	(5/6)(5/6)(1/6)	(5/6)(5/6)(5/6)(1/6)
or	.16667	.13889	.11574	.09645

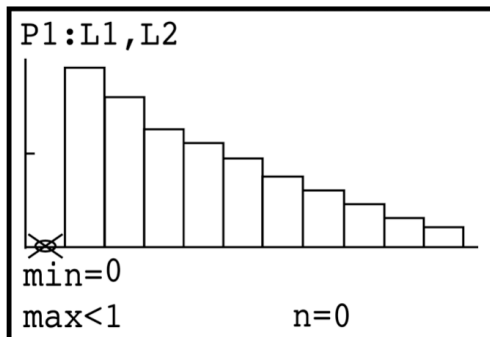


L1	L2	L3	2
1	.16667	-----	
2	.13889		
3	.11574		
4	.09645		
5	.08038		
6	.06698		
7	.05582		

L2(1) = .1666666666...

Using the calculator:

Create L1 with values for X (use 1-10)
highlight L2 header and select:
2nd | VARS | geometpdf(1/6,L1)
and then hit enter.



Now generate a histogram with L1,L2

Calculating the mean (expected value)

X	1	2	3	4	...
P(X)	p	qp	q ² p	q ³ p	...

$$\mu_x = (1)(p) + (2)(qp) + (3)(q^2p) + (4)(q^3p) + \dots$$

$$\mu_x = p + 2qp + 3q^2p + 4q^3p + \dots$$

$$q\mu_x = qp + 2q^2p + 3q^3p + 4q^4p + \dots \quad \text{multiply both sides by } q:$$

$$\mu_x - q\mu_x = p + qp + q^2p + q^3p + \dots \quad \text{subtract the two equations}$$

$$\mu_x(1 - q) = p + qp + q^2p + q^3p + \dots \quad \text{sum of infinite series} = 1$$

$$\mu_x(p) = 1$$

$$\mu_x = 1/p$$

$$\mu_x = 1/p$$

$$\sigma_x^2 = q/p^2$$

$$\sigma_x = \sqrt{q}/p$$

Example:

If the probability of success (rolling a 2) is $1/6$, and we know the distribution is geometric (ex. 8.15), then the Expected value for number of rolls to get a 2 is:

$$\begin{aligned}\mu_x &= 1/p \\ &= 1/(1/6) = 6\end{aligned}$$

You would expect to roll the dice 6 times to get a 2

If we want to know the probability that it takes more than 6 rolls to get a 2, then:

$$\begin{aligned}P(X>6) &= (1-p)^n \\ &= (5/6)^6 \\ &\approx .335\end{aligned}$$

$$P(X>n) = q^n$$